Liquid metal anode X-ray tubes and their potential for high continuous power operation

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Abstract

A novel type of electron-impact X-ray source is described in which X-rays are produced in a turbulently flowing liquid metal that is separated from the vacuum region of the X-ray source by a thin membrane. Following a summary of the physics of electron and photon transport applicable to the liquid metal anode X-ray (LIMAX), the three diffusion processes responsible for thermal transport in (electron diffusion, heat conduction and turbulent mixing) are briefly discussed and their relative importance is quantitatively assessed.

A simple Gaussian model is presented allowing the characteristic ranges of the three diffusion processes to be combined into a mean total diffusion range. The extent to which heat diffuses in the time taken for the liquid metal stream to pass the electron focus permits the loadability (electron beam power density per unit maximum anode temperature rise) of the turbulently flowing liquid metal target to be assessed.

A description of an experimental LIMAX facility constructed in these laboratories is given. Preliminary experimental results from the LIMAX facility are presented and satisfactory agreement is found between these results and the predictions of the Gaussian model for the mean total diffusion range. The suitability of the LIMAX proposal for X-ray scatter imaging investigations is briefly considered.

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1. Introduction

The field of X-ray scatter imaging is concerned with the unification of the image-forming and the material analysis capabilities of X-rays. The latter includes such physical phenomena as X-ray diffraction (XRD), Compton profile analysis, Rayleigh-to-Compton ratio method and X-ray fluorescence (XRF). X-ray scatter imaging has been successfully applied to the detection and identification of plastic explosives in suitcases (Luggar et al., 1995; Speller et al., 1993; Strecker et al., 1993; Yxlon, 1999) and it is being actively pursued for the characterisation of breast tumours (Rogers et al., 1999; Fernandez et al., 2002), to mention two examples.

Reference may be made to review articles (Harding, 1997; Harding and Schreiber, 1999) for applications of elastic and inelastic X-ray scatter imaging in biomedical science and industry.

The need is often felt in scatter imaging work for a compact and convenient X-ray source which can be continuously operated at high power (several tens of kW). This requirement excludes stationary and rotating anode X-ray tubes whose continuous power rating for focal spot sizes of 1 mm or so is restricted to below 10 kW. In this article, we discuss a novel type of electron impact source which has the potential for fulfilling the above need and which thus could prove beneficial for X-ray-based analysis and imaging techniques.
2. Electron impact X-ray generation

When high-energy electrons penetrate material, they lose energy by collisional excitation involving the atomic electrons (ionisation) and occasionally, by production of bremsstrahlung radiation in the nuclear Coulomb field. The ratio of the cross-sections for radiation emission to collisional excitation increases both with the electron energy and the atomic number (Heitler, 1964) but is substantially less than 1% even for large Z over the range of electron energies applicable to diagnostic radiology. A significant technological problem associated with the design of high-power electron impact X-ray sources is thus that of dissipating the heat produced in the target by the electron beam.

All electron impact X-ray sources employ a thermal transport mechanism to convey heat from the target to a coolant. Stationary anode tubes mainly rely on thermal conduction from the anode to coolant medium, whereas rotating anode X-ray tubes generally employ a combination of radiation transport and thermal conduction. Radiation transport from the anode to the X-ray tube housing requires the anode to possess a large anode surface area and to operate at a high temperature (radiated power \( \propto T^4 \)) if significant power levels are to be achieved. The radiation component of thermal transport rarely exceeds a few kW in practice, even for anodes operating at \( \sim 1000°C \).

Thermal transport by conduction implies similar limitations. A certain temperature offset is required to drive power from the anode into the coolant through the thermal resistance of the anode body, bearings, etc. As the maximum operating temperature of the anode is restricted (ultimately by the onset of melting), the temperature rise in the focus induced by the electron beam is itself limited. Typically, the anode surface in the vicinity of the focus must have a temperature offset of \( \sim 1000°C \) in order to drive a power of several kW into the coolant.

3. Liquid metal anode X-ray source

The suggestion has recently been made (Harding, 2001) of producing X-rays by electron bombardment of a turbulently flowing liquid metal which also acts as the coolant medium. The acronym liquid metal anode X-ray (LIMAX) will be used below for this concept. It was envisaged in this suggestion that the electron beam would be produced in a vacuum, separated by a “thin” electron window from the liquid metal. Thus the high-energy electron beam (\( \sim 150\) kV) penetrates the electron window, losing as little energy in the window as possible, before entering the liquid metal to produce X-rays.

A schematic illustration of the LIMAX concept is shown in Fig. 1. High negative voltage is applied through an insulating body to the cathode of the tube which, under the action of appropriate electron optics, yields a focussed electron beam. The beam is accelerated through the vacuum chamber onto the grounded electron window. The electrons are assumed to lose only a small fraction of their energy in the window. The electron beam diffuses into a liquid metal which is forced by the action of a pump at high speed into a constriction immediately below the electron window. Heat liberated in the anode is transported vigorously out of the interaction region by the liquid metal flow. Hot liquid from the focus is transported to a heat exchanger, where heat is extracted and the fluid is returned to its “cold” condition. The liquid is circulated back to the pump before returning to the interaction region with the electron beam. As the liquid metal anode can be directly cooled at the heat exchanger, the electron beam always interacts with “cold” fluid. This arrangement is similar in principle to a rotating anode X-ray tube, but with a geometry which allows the anode to be effectively cooled before it comes round to the electron beam again. For this reason, it is postulated that the LIMAX tube should have a high DC thermal loadability.

The electron window is clearly a critical component having to withstand both high pressures and
temperatures while being relatively transparent to electrons. Membranes of Ti foil having thickness \( \sim 1 \mu m \) and capable of sustaining differential pressures of several bar over an unsupported width of 1 mm have been reported (Ulrich et al., 2000) in connection with electron beam excitation of dense gases. The question of the feasibility of the LIMAX concept depends above all on the technological realisability of the electron window. It is outside the scope of this article to consider this question further here.

Granted the availability of such membranes, an X-ray tube of the suggested design would have the attractive property of directly depositing the electron beam energy into the coolant and thereby avoiding the thermal transport problem of conventional stationary or rotating anode X-ray tubes. Liquid metals have much lower thermal conductivities than their solid-state counterparts and in the next section we address the question of the thermal loadability arising from the use of liquid rather than solid metals.

4. Thermal loadability of LIMAX

In this section, a simple estimate will be presented of the thermal loadability, \( L \), of the LIMAX concept, where \( L \) has units (W mm\(^{-2}\)K\(^{-1}\)) of electron beam power per unit focus area and unit temperature increment at the hottest point, i.e. where fluid exits the focus region.

Referring to Fig. 2 (an enlarged view of the interaction region of Fig. 1), it is assumed that an electron beam travelling vertically downwards (in the positive Z direction) interacts with a liquid flowing with main stream velocity \( U_0 \) in the X direction. The membrane is assumed to lie in the XY plane. If a line focus electron beam is employed to maximise the total power while maintaining a small (projected) focal spot, the narrow dimension of the line (\( F_x \sim 1 \) mm) is in the X direction whereas its long dimension (\( F_y \sim 10 \) mm) is in the Y direction. Values were given in Harding (2001) for the height, \( d \), of the interaction cavity as 50 \( \mu m \) and \( U_0 \sim 500 \) m s\(^{-1}\).

4.1. Liquid metal selection

As the liquid metal plays such an important role in the LIMAX arrangement both as the medium in which X-rays are produced and also as the coolant which removes heat from the focus region, a brief description will now be given of the major thermophysical and hydrodynamical properties of a selection of liquid metals.

There are several desirable properties for a liquid metal to possess in the present case including a relatively high atomic number for X-ray bremsstrahlung production, a melting point around room temperature and ease of availability. In addition to the element Hg which is liquid at room temperature and Ga which melts at 29.8\(^\circ\)C, there are several indium alloys of which the ternary eutectic GaInSn (wt\%: Ga 62.5, In 21.5, Sn 16) and the quaternary eutectic BiPbInSn (wt\%: Bi 49.4, In 21.0, Pb 18.0, Sn 11.6) are worthy of mention. Some physical properties of these alloys are listed in Table 1 together with the properties of Hg for comparison.

It should be noted that, with the exception of the first row of Table 1, these values are temperature-dependent and apply just above the melting points. Using mercury as an example (Incropera and De Witt, 1996), the density, heat capacity and kinematic viscosity decrease with increasing temperature, in the case of the viscosity by over 40% from 300 K to 600 \(^\circ\)K. By contrast, the thermal conductivity and thermal diffusivity increase with increasing temperature, in the latter case by 43% over the above temperature range. Similar behaviour holds for the other candidates in Table 1, the viscosity of GaInSn, e.g., dropping to half its melting point value at 200\(^\circ\)C.

4.2. Reynolds number

The Reynolds number (Schlichting, 1979) relates the inertial force to friction force acting on a fluid particle and is widely used to characterise liquid flow:

\[
Re = \frac{U_0d}{v}
\]

This has the numerical value of \( Re \sim 10^5 \) for the above values for \( U_0 \) and \( d \) in the arrangement depicted in Fig. 2. The transition from laminar to turbulent flow in the arrangement of Fig. 1 depends on such details as wall roughness, flow history before entering the constriction, etc. The liquid flow in this case is expected to be turbulent and can thus be divided into near-wall and far-wall regimes. In the former there exists a boundary layer in which laminar flow obtains, whereas further from the wall the liquid rapidly attains its main stream...
velocity. The thickness, $\delta$, of the laminar boundary layer is given (Schlichting, 1979) as

$$ \delta \approx \sqrt{\frac{\nu l}{U_0}}. \quad (2) $$

where $l$ is the wall length on which the boundary layer builds up in the flow direction. $\delta$ has the numerical value of $\sim 5 \mu m$ in the present case.

### 4.3. Thermal transport processes in the LIMAX tube

From the foregoing discussion, it is evident that there are three separate physical processes influencing the transport of heat. These are: the diffusion of high-energy electrons into the liquid which terminates when their energy is liberated in the form of heat; thermal conduction in the liquid metal mediated by the conduction electrons; and turbulent mixing of the liquid metal in the region of the membrane/liquid interface due to mass transport in a transverse direction to that of the mean flow. These effects will be described in the following sections.

#### 4.3.1. Electron range

The transport of high-energy electrons in material of moderate to high $Z$ is often described as a diffusion process. The path of the electrons is tortuous and consists of occasional large deflections (nuclear elastic scattering) separated by approximately linear tracks along which the electron loses energy to its surroundings. The integrated path length (Dyson, 1990) is the mean total distance covered by the electron along its path. This has a value of $\sim 0.55 \text{ kg m}^{-2}$ for 150 keV electrons traversing moderate to high $Z$ elements (Berger and Seltzer, 1964). The projected electron range, i.e. the mean projection of the electron paths in the direction of incidence is often described by the Thomson–Whiddington law (Dyson, 1990) which is itself derived from the Bethe–Bloch energy loss relationship (Dyson, 1990). The Thomson–Whiddington law is

$$ x = \frac{(E_0^2 - E^2)}{b\rho}. \quad (3) $$

$E_0$ is the initial electron energy and $x$ is the projected range required to reduce the mean electron energy to $E$. The other symbols have their customary meanings. The Thomson–Whiddington constant, $b$, has a value for tungsten of $8.5 \times 10^4 \text{ keV m}^2 \text{ kg}^{-1}$ for large $Z$ at 150 keV, implying a value for $(x)_{0.27 \text{ kg m}}^{-2}$ of $0.27 \text{ kg m}^{-2}$. For a liquid of density $8.2 \times 10^3 \text{ kg m}^{-3}$, a projected range of $32 \mu m$ results.

The projected energy loss per unit distance in the $Z$ direction, parallel to the original propagation direction of the electron beam, has a maximum at the so-called diffusion depth but for the purposes of this communication, it will be assumed that energy is uniformly absorbed over the mean projected range.

#### 4.3.2. Thermal diffusion distance

On the basis of the values for $U_0$ and $F_x$ given in Section 4, the mean time taken for the liquid to flow past the focus is $\sim 20 \mu s$. It will be demonstrated below that the depth in the $Z$ direction to which heat diffuses in this time interval is much less than the focal spot size in the $X$ and $Y$ directions. Hence only the diffusion of heat in the $Z$ direction (parallel to the electron beam) need be considered. It is a standard result of statistical mechanics (Reif, 1965) that the root mean square distance, $\tilde{A}$, from the origin to which particles diffuse in a time $t$ is

$$ \tilde{A} = \sqrt{2ht}. \quad (4) $$

In Eq. (4), $h$ is the thermal diffusivity ($\text{m}^2 \text{s}^{-1}$) and $t$ will be equated with the time $(= F_x / U_0)$ taken for the liquid to cross the electron beam. $\tilde{A}$ from Eq. (4) is $20 \mu m$. It is interesting to note that the relative magnitudes of the rms heat diffusion distance and the projected electron range are similar, indicating the importance of taking both factors into account.

#### 4.3.3. Mixing length

It was emphasised above that the flow of liquid metal in the interaction region is characterised by a high degree of turbulence, having a Reynolds number greater than $10^5$. Under these conditions, the inertial force which fluid particles exert on their surroundings is much greater than the viscous force which damps movement.
perpendicular to the flow direction. There is hence a diffusion of cold fluid particles from the main stream into the interaction region of the electron beam and conversely a diffusion of hot fluid particles into cold regions of the liquid. Both processes are to a certain extent independent and tend to increase the volume of fluid in which heat energy is deposited. Turbulent mixing thus acts to lower the temperature of the fluid. This increased heat diffusion via mass transport in a turbulent flowing liquid is without parallel in conventional stationary or rotating anode X-ray tubes.

In turbulent flow conditions, fluid particles of transitory existence are observed which are continually forming and being destroyed. It is common to talk of these fluid particles as randomly colliding with one another, in analogy with the kinetic theory of gases. Continuing the analogy with gas molecules, the movement of these fluid particles can be characterised by a mean free path. In turbulent mixing theory, this mean free path is identified with the “mixing length” which was originally introduced by Prandtl (see Schlichting, 1979; p. 580).

The Prandtl mixing length theory leads to conclusions which have been subjected to extensive experimental tests. For all incompressible fluids with \( Re \geq 10^5 \) it transpires that there is a simple relationship between mixing length, \( L \), and transverse distance (\( Z \) in Fig. 2) from the wall as follows:

\[
L = \chi Z. \tag{5}
\]

In words, the mixture length tends to zero in the neighbourhood of the wall and increases linearly with distance from it. Since the liquid viscosity is an expression of the intermolecular potential, and a large value of \( Re (\geq 10^5) \) corresponds to the situation where this is of negligible importance relative to the inertial energy, it is reasonable to expect the value of \( \chi \) to be independent of the nature of the liquid and \( Re \). Measurements confirm this expectation and imply a numerical value for \( \chi \) of \(~0.4\), independent of wall roughness, liquid viscosity, flow speed, vessel shape, etc. It is cautioned against taking the analogy between turbulent diffusion and kinetic processes in gases too far as the independence of \( \chi \) on the above parameters demonstrates.

4.4. Estimation of temperature–power characteristic

Three important diffusion mechanisms have been described which contribute to the thermal loadability of the X-ray device described here by increasing the effective volume in which thermal energy is deposited. The first, electron diffusion, is relatively independent of the anode material whereas the second, heat conduction, depends significantly on thermophysical anode characteristics. The final contribution, the turbulent mixing length is independent of the physical properties and nature of the liquid for \( Re \geq 10^5 \).

4.5. Gaussian approximation for temperature profile

Interest is attached in the present work to the expected loadability of a liquid metal anode X-ray tube. The authors are aware of the standard analytical solutions to the rotating anode loadability problem (Bouwers, 1927) including approximations for the electron penetration into the anode (Whitaker, 1988). It is however not obvious to the authors how to go about finding analytical solutions to the present problem which includes three diffusion processes of varying origins and characteristics.

Nevertheless, an impression of the temperature and power loadability may be gained in the following way. The temperature distribution \( T(Z) \) at \( X = X_c \) (the hottest point in the focus) is the result of three independent, uncorrelated, diffusion processes. Hence the most reasonable assumption for \( T(Z) \) is that it is of Gaussian form. If the diffusion of heat in the present situation is regarded as a 1D “random walk” problem (Haji-Sheikh and Sparrow, 1967), the Gaussian form is a good approximation for the temperature distribution for spatial ranges large compared with the mean free paths of the diffusion processes. As heat energy is introduced at the \( Z=0 \) boundary and cannot be dissipated in the negative \( Z \) direction, we expect the temperature distribution to have its maximum value, \( T_p \), there i.e the assumed Gaussian is truncated for \( Z<0 \). The authors have reassured themselves that the solution of this “frustrated random walk” problem is indeed a truncated Gaussian.

In first order, the standard deviation, \( \sigma \), of the assumed Gaussian profile will be the result of combining in quadrature the characteristic lengths, \( \sigma_{\text{process}} \), of the component diffusion processes of Section 4. in the following manner:

\[
\sigma_{\text{total}} = \sqrt{\sigma^2_{\text{electron}} + \sigma^2_{\text{heat}} + \sigma^2_{\text{mixing}}}. \tag{6}
\]

The first of these, \( \sigma_{\text{electron}} \), is identified with the electron range discussed in Section 4. 3.1. The second, \( \sigma_{\text{heat}} \), (rms heat diffusion length) has also been evaluated quantitatively in Section 4.3.2. The third process, turbulent mixing, is without analogy in conventional solid state X-ray targets. We evaluate a mean mixing length, \( \sigma_{\text{mixing}} \), from Eq. (5) evaluated at a \( Z \) co-ordinate corresponding to the combination in quadrature of the rms depths of the electron and thermal diffusion processes.

It has to be emphasised that the choice made here of a truncated Gaussian temperature distribution is entirely heuristic. The question as to the suitability of this choice can only be answered when measurement data or a
Table 2
RMS diffusion depths for two liquid metals

<table>
<thead>
<tr>
<th>Material</th>
<th>GaInSn</th>
<th>BiPbInSn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron range (μm)</td>
<td>41</td>
<td>32</td>
</tr>
<tr>
<td>Diffusion heat depth (μm)</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>Mean mixing length (μm)</td>
<td>27</td>
<td>19</td>
</tr>
<tr>
<td>RMS diffusion depth (μm)</td>
<td>55</td>
<td>39</td>
</tr>
</tbody>
</table>

reliable finite element simulation program, incorporating turbulence, heat conduction and the finite electron range are available.

4.6. RMS diffusion depth for two liquid metals

Material properties for two liquid metals of interest, GaInSn and BiPbInSn, were presented in Table 1. The characteristic lengths of the three diffusion processes described in the preceding section have been evaluated on the basis of the appropriate equations. These are Eq. (3) for the projected range, Eq. (4) for the rms heat diffusion length and Eq. (5) for the turbulent mixing length.

These lengths were combined as described in Section 4.5 and the results are presented in Table 2. The ranges of the three processes are of the same order of magnitude, implying that each must be considered, and none can be neglected, if reliable temperature estimates are to be obtained.

4.7. Calculation of maximum temperature

We assume as described above a Gaussian temperature profile \( T(Z) \) in the centre of the line focus \( (Y = 0) \) at the point \( X = F_Y \). The temperature at the inlet to the electron interaction region is assumed to be zero

\[
T(Z) = T_0 \exp \left\{ -\frac{Z^2}{\sigma^2} \right\} \quad \text{for } 0 \leq Z \leq \infty, \quad (7)
\]

\[
T(Z) = 0 \quad \text{otherwise.}
\]

It is further assumed that the liquid flow speed, \( U_0 \), is independent of \( Z \) apart from in the laminar boundary layer whose thickness can be neglected in comparison with the diffusion depths indicated in Table 2. The temperature in an elemental layer of liquid at depth \( Z \) is proportional through the heat capacity to the energy stored in that layer. The integral of temperature over depth \( Z \) (Eq. (8)) is therefore proportional to the total thermal energy imparted per unit time to the fluid

\[
\int_{0}^{\infty} T(Z) \, dZ = T_0 \frac{\sqrt{\pi}}{2\sigma}. \quad (8)
\]

The electron beam power, \( P \), is simply the product of thermal capacity \( (c_p) \), effective heat depth \( (1/2\sigma \sqrt{\pi}) \) the length of fluid heated by the focus per second \( (U_0) \) and its width perpendicular to the flow direction \( (F_Y) \) with the temperature increment, \( T_0 \). Hence we derive:

\[
P = \frac{T_0 \sigma \sqrt{\pi} c_p U_0 F_Y}{2}. \quad (9)
\]

A value for the power density, \( P \), of 1 kW mm\(^{-2}\) is assumed, implying a total electrical power of 10 kW in a 10 mm \( \times \) 0.1 mm focus. Insertion of the rms diffusion depths from Table 2 into Eq. (9) and using the values introduced above for the other variables allows \( T_0 \) to be calculated. The result for GaInSn is \( T_0 \sim 180^\circ \) and for BiPbInSn is \( T_0 \sim 240^\circ \). It appears from this simple model that the LIMAX principle might permit a DC electrical power of several tens of kW for an electron beam having a line focus of dimensions \( \sim 1 \times 10 \text{ mm}^2 \). Hence the liquid metal anode technology appears to represent a significant improvement in DC loadability relative to stationary anode X-ray tubes, which are limited to a few kW power for this focus size.

The contributions to thermal loadability of electron range and thermal diffusivity depend solely on the choice of anode material as discussed in Sections 4.3.1 and 4.3.2. The turbulence contribution within the boundary layer can be increased, as discussed by Schlichting (1979), in several ways. These include 2D (e.g. trip wire) and 3D (e.g. cup) structures and distributions of these structures located at the boundary wall. Moreover, the flow along concave surfaces leads to instabilities in the boundary layer (e.g. Taylor–Goertler vortices) which further promote thermal transport. Turbulent mixing considered in Section 4.3.3 may prove to be a significant additional contribution to the thermal transport mechanisms of LIMAX relative to conventional solid-state electron impact X-ray sources.

5. Experimental results

5.1. Description of apparatus

An experimental LIMAX facility has been constructed in the Philips Research Laboratory, Hamburg. A standard single pole (grounded anode) generator supplies high voltage regulated to a stability of better than 1% to the insulated cathode. The cathode together with its electron optics allows a 150 keV electron beam of 1 mm width and having a length of several mm to be focussed onto an anode module incorporating the electron window. Metal foils (e.g. Ti, Mo) of several μm thickness have been used as the electron window. After passing through the window, the electron beam interacts with a stream of liquid metal (gallium indium tin alloy) flowing at a mean speed of up to several tens of metres per second. The maximum temperature of the foil is measured with infrared thermography. The fluid was usually held at room temperature by the action of a
water-cooled cross-flow heat exchanger and maximum temperature increments in the focus of a few hundred degrees were registered. The loadability is derived as the power density of the electron beam normalised to the maximum temperature rise it induces in the liquid metal stream.

A representative set of results is reproduced in Fig. 3. The quantitative and qualitative agreement between the predictions of the simple model presented here and the measured data are encouraging and probably fortuitous, in view of the number of simplifying assumptions in our model. No attempt has been made to promote turbulence in the region of flow where the electron beam interacts with the liquid metal.

6. Conclusion

A calculation of temperature rise in the focus of a liquid metal anode X-ray tube (LIMAX) has been performed, based on an assumed Gaussian form of temperature profile. Three diffusion processes play a role in the transport of heat and their spatial ranges are similar to one another in the case considered. The temperatures derived represent order of magnitude estimates as they include assumptions, mainly the Gaussian temperature distribution (Eq. (7)) which can only be verified by accurate measurement or numerical analysis.

Satisfactory agreement has been obtained between the predictions of the Gaussian model and preliminary results for the dependence of loadability on fluid velocity measured in an experimental LIMAX facility. These results should be extended to other working fluids besides the GalnZn alloy considered here and to a greater range flow speeds.

No particular effort has been made in this work to optimise the contribution of turbulence to thermal transport. The role of turbulent mixing is doubtless worthy of further investigation for several reasons: it has a significant effect on the results, it is without parallel in conventional X-ray tubes and it could be enhanced in ways suggested in Section 4.7 to further improve the power loadability of this type of X-ray tube.

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